Test of Time Reversal Symmetry in the Proton Deuteron System

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Abstract

Internal target experiments with high quality proton beams allow for a new class of experiments providing null tests of time reversal symmetry in forward scattering. This could yield more stringent limits on T-odd P-even observables. A excellent candidate for such experiments is the proton deuteron system. This system is analyzed in terms of effective T-violating P-conserving nucleon-nucleon interactions and bounds on coupling strengths that might be expected are given.

1. Introduction

The existence of CP-violation is well established through decays in the K_L, K_S system [1]. Despite strong efforts, experiments on other systems have given only bounds on CP-violating and on T-violating interactions. Both symmetries are treated equivalently in the following due to no experimental evidence of CPT-violation.

Among these efforts are the measurements of the electric dipole moments of the neutron [2,3], the atom and the electron [4], T-odd correlations in β-decay [5] and γ angular distributions [6], compound-nucleus reactions as well as tests of detailed balance [7-9]. For a review on more experiments see refs. [10-14].

It is important to distinguish P-violating (P-odd) from P-conserving (P-even, viz. C-odd) breaking of time reversal symmetry. It is only the P-odd violation of CP that has been established and accommodated by the Kobayashi-Maskava matrix. To date there is no experimental evidence for P-even violation of CP symmetry. Also the question whether the standard model alone provides any T-odd P-even interaction on tree level, is presently all under discussion [15,16].

Experiments in nuclear systems are usually analyzed in terms of effective T-odd nucleon-nucleon (NN) interactions [17,18]. This seems a reasonable parameterization for the moderate energies involved in most experiments. For complex nuclei similar to the treatment of P-violation [19], effective T-odd one particle potentials have been introduced [20,21]. Due to the complex structure of nuclei, enhancement factors may occur that lead to advantageous experimental observables [20,22-25]. Many examples of such an enhancement have been found in the context of parity violation experiments such as in ^{180}Hf γ-correlation or ^{139}La thermal neutron transmission, for a review see e.g. [26]. Therefore, similar experiments to test time reversal symmetry have been suggested [24, 27].

Unfortunately, enhancement factors of this kind seem absent for light nuclear systems, compare also [18]. On the other hand, the proton deuteron (pd) system considered here would fully utilize the high luminosity of high quality proton beams combined with internal target techniques. A forward scattering experiment would provide a null test of time reversal symmetry, as

pointed out by Conzett recently [28]. Other setups for scattering processes (two particles in two particles out) would not lead to a true null test, in the sense of measuring a unique T-odd observable [29]. Forward scattering could lead to an experimental accuracy of $|\langle T - \text{odd}\rangle| < 10^{-6}|\langle T - \text{even}\rangle|$ [30]. This accuracy has been unmatched before and therefore this system will be analyzed in the following.

2. T-odd Correlations and Forward Scattering

The optical theorem relates the total cross section to the forward scattering amplitude $\mathcal{F}$ via (final state polarization not observed) [32]

$$\sigma_{tot} = 4\pi/k \ \mathrm{Im}\, tr(\mathcal{F}\rho)/tr(\rho) \tag{1}$$

For convenience the initial state density matrix ρ is expanded in terms of product spin tensors

$$\left[S_1^{[\lambda]} \otimes S_2^{[\kappa]}\right]_M^{[J]} = \sum_{\nu\mu} \langle \lambda\mu\kappa\nu \,| JM \rangle \, S_{1,\mu}^{[\lambda]} S_{2,\nu}^{[\kappa]} \tag{2}$$

with $S_{1,\mu}^{[\lambda]}(S_{2,\nu}^{[\kappa]})$ spin tensor for particle 1(2) [31, 33], of rank $\lambda(\kappa)$ and the Clebsch-Gordan coefficient $\langle \lambda\mu\kappa\nu \,| JM \rangle$ [31]. The spin tensors are normalized, in accordance with the Madison convention [33], such that the reduced matrix element is given by $\left\langle j \left\| S^{[\lambda]} \right\| j \right\rangle = \hat{j}\hat{\lambda}$, where $\hat{j} = (2j+1)^{1/2}$. The density matrix then reads ($S^{\dagger\lambda}$ denotes the hermitian conjugate)

$$\rho = tr(\rho)(\hat{j}_1\hat{j}_2)^{-2} \sum_{\lambda\kappa} \sum_{JM} \mathrm{t}_{\lambda\kappa,M}^{[J]} \left[S_1^{\dagger[\lambda]} \otimes S_2^{\dagger[\kappa]}\right]_M^{[J]} \tag{3}$$

with $\mathrm{t}_{\lambda\kappa,M}^{[J]}$ denoting the tensor moments, viz.

$$\mathrm{t}_{\lambda\kappa,M}^{[J]} = \sum_{\nu\mu} \langle \lambda\mu\kappa\nu \,| JM \rangle \, \mathrm{t}_{\lambda\mu,\kappa\nu} = \sum_{\nu\mu} \langle \lambda\mu\kappa\nu \,| JM \rangle \, \mathrm{t}_{1,\mu}^{[\lambda]} \mathrm{t}_{2,\nu}^{[\kappa]} \tag{4}$$

The last equality in eq. (4) holds for uncorrelated incoming particles.

In the above spin basis eq. (2) and in the center of mass system (i.e. $\mathbf{k} = (\mathbf{p}_1 - \mathbf{p}_2)/2, \mathbf{p}_1 + \mathbf{p}_2 = 0$) the forward scattering amplitude $\mathcal{F}$ may be

decomposed as follows

$$\mathcal{F} = \sum_{\lambda\kappa} \sum_{J} \mathcal{F}_{\lambda\kappa;J} \left[\left[S_1^{[\lambda]} \otimes S_2^{[\kappa]} \right]^{[J]} \otimes k^{[J]} \right]^{[0]} \tag{5}$$

with

$$k_M^{[J]} = \left[\frac{4\pi J!}{(2J+1)!!} \right]^{1/2} i^L Y_{JM}(\hat{k}) \tag{6}$$

where $\hat{k} = \mathbf{k}/k$. Note, that for $\mathbf{k} = k\hat{e}_z$ (Madison convention) only moments with $M = 0$, can contribute to the total cross section (rotational symmetry of forward scattering). Evaluating the spin trace, viz.

$$tr\left(\rho \ \left[\left[S_1^{[\lambda]} \otimes S_2^{[\kappa]} \right]^{[J]} \otimes k^{[J]} \right]^{[0]} \right) = tr(\rho) \left[\mathrm{t}_{\lambda\kappa}^{[J]} \otimes k^{[J]} \right]^{[0]} \tag{7}$$

the total cross section can be written as

$$\sigma_{tot} = 4\pi/k \sum_{\lambda\kappa} \sum_{J} \mathrm{Im} \left(\mathcal{F}_{\lambda\kappa;J} \left[\mathrm{t}_{\lambda\kappa}^{[J]} \otimes k^{[J]} \right]^{[0]} \right) \tag{8}$$

This is the most general form of the total cross section, if final state polarization is not observed. If $\kappa = 0 (\lambda = 0)$ and therefore $\lambda = J (\kappa = J)$ in eq. (8), then particle 2(1) is not polarized and for $J \neq 0$ the forward scattering amplitude $\mathcal{F}_{J0;J}$ ($\mathcal{F}_{0J;J}$) depends on the polarization of one particle only. If $\kappa \neq 0$ and $\lambda \neq 0$, then the total cross section depends upon the correlation between the spins of the two incoming particles.

To exhibit the symmetry relations of $\mathcal{F}_{\lambda\kappa;J}$, the forward scattering amplitude will now be decomposed according to parity and time reversal symmetry. This follows from the properties of the spin tensors and the spherical harmonics under parity and time reversal symmetry [31], viz.

$$\mathcal{F}_{\lambda\kappa;J} = \frac{1}{4} \sum_{\eta\tau} (1 + \eta\tau(-1)^{\lambda+\kappa})(1 + \tau(-1)^{\lambda+\kappa+J}) F_{\lambda\kappa;J}^{\eta\tau} \tag{9}$$

with $\eta = 1(-1)$ for P-even(odd) and $\tau = 1(-1)$ for T-even(odd) symmetry. For $\lambda = \kappa = 0$, i.e. unpolarized initial particles, only $\eta = \tau = 1$ can contribute to the total cross section. Inserting $\lambda = 0$, $\kappa = J$ or $\kappa = 0$, $\lambda = J$ and $\tau = \pm 1$ in eq. (9), one finds that in forward scattering only

Table 1: Symmetry relations implied on the forward scattering amplitude via eq.(9)

	J even	J odd
$(\lambda+\kappa)$ even	E	TP
$(\lambda+\kappa)$ odd	T	P

T-even ($\tau = 1$) amplitudes are possible. This situation is different from pure P-violation ($\eta = -1$, $\tau = 1$ in eq.(9)), which means that a P-odd experiment can be conducted by polarizing only one of the participating particles (projectile or target). To test for time reversal symmetry with a single measurement one needs to have both particles polarized and measure a correlation of the spins. This can be achieved by setting $\kappa \neq 0$ and $\lambda \neq 0$ in eq.(9). Then there is still the choice to distinguish between P-even (J even) or P-odd (J odd) violation of time reversal symmetry. This way all possible observables for which final state polarizations are not observed are exhausted. In the following a notation for $\mathcal{F}^{\eta\tau}_{\lambda\kappa;J}$ that exhibits the symmetry relations more obviously is used, viz. $\mathcal{F}^{E}_{\lambda\kappa;J}$, $\mathcal{F}^{P}_{\lambda\kappa;J}$, $\mathcal{F}^{T}_{\lambda\kappa;J}$, $\mathcal{F}^{TP}_{\lambda\kappa;J}$ for the T-even P-even (E), T-even P-odd (P), T-odd P-even (T), T-odd P-odd (TP) forward scattering amplitude. See also table 1.

For spin-$\frac{1}{2}$ particles ($j_1 = j_2 = \frac{1}{2}$), the following amplitudes contribute to forward scattering: $\mathcal{F}^{E}_{00;0}$, $\mathcal{F}^{P}_{10;1}$, $\mathcal{F}^{P}_{01;1}$, $\mathcal{F}^{E}_{11;0}$, $\mathcal{F}^{E}_{11;2}$, $\mathcal{F}^{TP}_{11;1}$. Note that there is no T-odd P-even forward scattering amplitude $\mathcal{F}$. For the more general case with final momentum **k**'$\neq$**k**, all possible spin combinations have been given by Wolfenstein [34].

For the pd-system with $j_1 = \frac{1}{2}$ and $j_2 = 1$, the amplitude has a much richer structure. Beside the amplitudes mentioned above additional spin contributions are possible: $\mathcal{F}^{E}_{02;2}$, $\mathcal{F}^{P}_{12;1}$, $\mathcal{F}^{P}_{12;3}$, $\mathcal{F}^{T}_{12;2}$. For the more general case **k**$\neq$**k**' the amplitudes have been given by Seyler [35]. The symmetry assignments can be easily reproduced with help of table 1. For this system in particular a T-odd P-even amplitude is present, which would be zero, if time reversal invariance holds. An experiment sensitive to $\mathcal{F}^{T}_{12;2}$ would be a true null test of time reversal invariance.

To be more specific, the tensor moments $\mathrm{t}^{[J]}_{\lambda\kappa}$, eq. (8), of the incoming

particle may be chosen such that the total cross section for spin-1/2 spin-1 scattering is given through ($\mathbf{k}$=$k\hat{e}_z$)

$$\sigma_{tot}^T = \sigma_{tot}^0 - \frac{4\pi}{k}\sqrt{\frac{2}{15}}\,\mathrm{Im}\left(\mathcal{F}_{12;2}^T \mathrm{t}_{12}^{[2]}\right) \tag{10}$$

with σ_0 the total unpolarized cross section. The tensor moment $\mathrm{t}_{12}^{[2]}$ may be rewritten in terms of cartesian coordinates, by using eq. (4) for uncorrelated inital states and rewriting $\mathrm{t}_{\pm 1}^{[1]} = \mp(P_x \pm iP_y)/\sqrt{2}$ and $\mathrm{t}_{\pm 1}^{[1]} = \mp(P_{xz} \pm iP_{yz})/\sqrt{3}$ Here, P_y and P_x denote the polarizations and P_{xz}, P_{yz} the alignment of the initial particles with respect to the cartesian basis, viz.

$$\mathrm{t}_{12}^{[2]} = -i(P_{xz}P_y - P_{yz}P_x)/\sqrt{3} \tag{11}$$

Note, that due to rotational symmetry of forward scattering the cartesian coordinate system may be rotated around the z-axis, such that $P_{yz}P_x = 0$. Inserting eq. (11) into eq. (10), a T-odd P-even "total cross section asymmetry" or "total spin correlation" $\mathcal{S}^T$ may then be defined through

$$\sigma_{tot}^T = \sigma_{tot}^0(1 + P_{xz}P_y\mathcal{S}^T) \tag{12}$$

which is explicitly given by

$$\mathcal{S}^T = \frac{4\pi}{3k}\sqrt{\frac{2}{5}}\,\mathrm{Re}(\mathcal{F}_{12;2}^T)/\sigma_{tot}^0 \tag{13}$$

Note that being T-odd, the correlation $\mathcal{S}^T$ is sensitive to the *real* part of the forward scattering amplitude $\mathcal{F}_{12;2}^T$, since $\mathrm{t}_{12}^{[2]}$ is imaginary, eq. (11). Eq. (13) will be evaluated in the following for the proton-deuteron system. Note, that the tensor structure $\left[\left[S_1^{[1]} \otimes S_2^{[2]}\right]^{[2]} \otimes k^{[2]}\right]^{[0]}$ implied here is present in the alternative expression $(\underline{\sigma} \times \mathbf{J} \cdot \mathbf{k})(\mathbf{J} \cdot \hat{k})$ with $\underline{\sigma} = \mathbf{S}_1$ and $\mathbf{J} = \mathbf{S}_2$, sometimes used in this context.

The dependence on the polarization of the incoming particles eqs. (10),(12) is unique for a T-odd P-even observable. The asymmetry may be extracted from the total cross section in a transmission experiment by switching the sign of $P_y(P_{xz})$ while keeping $P_{xz}(P_y)$ constant and subtract the transmission factors corresponding to the change of sign [28,30].

3. Nucleon Nucleon Amplitudes

Now the important question arises, which types of two nucleon amplitudes t_{NN} will lead to a T-odd P-even observable in the pd-system. In general also three body forces could contribute. However, though they may be present they are presumably not the dominant forces and are neglected in the following.

Due to the energy regime considered here, the question raised above may be answered in lowest order rescattering series. Indeed, comparison of experiments show that the total pd cross section is roughly the sum of neutron proton and proton-proton total cross sections [36], with sufficient accuracy for the present investigation.

As an example the basic mechanisms will be demonstrated on the pd break-up cross section in some detail. For simplicity I use channel notation for the three nucleon system, i.e. $t_k := t_{NN}(ij)$ with ijk permutation of particles 123, and following ref. [37] the break-up cross section in lowest order rescattering approximation may be written as

$$\sigma_{b-up} = 4\frac{E_pE_d}{Ek}tr\left(\frac{\rho}{6}\int \left\langle \phi_1\mathbf{k}_1 \left| t_2^\dagger + t_3^\dagger \right| \phi_0^S \right\rangle \left\langle \phi_0^S \mid t_2 + t_3 \mid \phi_1\mathbf{k}_1 \right\rangle \delta(E-E_0)\right) \tag{14}$$

with $E = E_p + E_d$ the total scattering energy, E_p, E_d the proton and deuteron energies resp., E_0 the energy of the free three particle state ϕ_0^S that is properly symmetrized in one pair coordinate. Since all particles are identical, the initial state $|\phi_1\mathbf{k}_1\rangle$, with the deuteron wave function ϕ, has been chosen to be in channel 1 [37]. The integral runs over all continuous and discrete quantum numbers of the final state ϕ_0^S. The factor 1/6 takes account of the 6 fold overcounting due to the symmetry of the final state. Note, that due to the rank of the spin tensor $S_2^{[2]}$ appearing in the T-odd observable $\mathcal{S}^T$ only channels with different indices, viz. $\propto \left\langle t_2^\dagger t_3 \right\rangle$, are nonzero. The direct channels viz. $\propto \left\langle t_2^\dagger t_2 \right\rangle$, $\left\langle t_3^\dagger t_3 \right\rangle$, are excluded by spin selection rules.

For $t_{NN} = t_{NN}^E + t_{NN}^T + t_{NN}^{TP} + t_{NN}^P$ eq. (14) separates into a sum of terms with different symmetry properties under T and P. Then the T-odd P-even total spin correlation $\mathcal{S}^T$, eq. (13) may arise through the following combinations

$$\mathcal{S}^T \quad \propto \quad \left\langle t_{NN}^T t_{NN}^E \right\rangle / \sigma_{tot}^0 \tag{15}$$

$$\mathcal{S}^T \quad \propto \quad \left\langle t_{NN}^{TP} t_{NN}^{P} \right\rangle / \sigma_{tot}^0 \tag{16}$$

Note that in the three body system both types, viz. P-even t_{NN}^P and P-odd t_{NN}^{TP}, violation of time reversal symmetry may contribute to a measurement of $\mathcal{S}^T$. They cannot be distinguished.

However, the bounds on t_{NN}^{TP} are rather stringent from electric dipole moment measurements [2,3,15,38], viz. $|\left\langle t_{NN}^{TP} \right\rangle| < 10^{-10}...10^{-11}|\left\langle t_{NN}^{E} \right\rangle|$. Also, the parity violating amplitude t_{NN}^P is expected be in the range of typical weak amplitudes, viz $|\left\langle t_{NN}^{P} \right\rangle| \simeq 10^{-7}|\left\langle t_{NN}^{E} \right\rangle|$ [29]. Therefore, the combination of such two body amplitudes would very likely lead to bounds of $|\left\langle t_{NN}^{TP} t_{NN}^{P} \right\rangle| \leq 10^{-17}\sigma_{tot}^0$ for the proton deuteron system. This value is far below the resolution that might be reached in a measurement of $\mathcal{S}^T$.

On the other hand, experimental bounds on the strength of the alternative combination $\left\langle t_{NN}^{T} t_{NN}^{E} \right\rangle / \sigma_{tot}^0$, eq. (15), which provides a test of generic T-violating P-conserving two body interactions, are much weaker. However, comparison of different experiments is rather difficult, which is mostly due to our lacking knowledge of a P-even breaking mechanism of time reversal symmetry. This is different from P-odd breaking, where mechanisms can be parameterized in terms of the standard model. In this sense each experiment is unique, and different experiments can only be compared by using even mild model assumptions to treat the dynamic behavior of the system in question. Before preceding, the bounds implied by these experiments will be discussed in the following.

Experiments on complex nuclei are usually analyzed in terms of effective one-body potentials with strength G^T , as suggested by Michel in the context of parity violation [19]. The probably most stringent limit from γ-decay comes from the experiment on ^{57}Fe [6]. It gives a bound through $G^T \simeq (0.7 \pm 1.6 \pm 0.5) \times 10^{-5}$[20], where the first error relates to the experimental error and the second to different residual interactions in the shell model analysis. Detailed balance experiments give bounds on T-odd P-even observables based on statistical interpretation of the level distribution of the compound nucleus. In a most recent analysis Harvey et al. give $G^T < 2.6 \times 10^{-4}$ [9]. However, bounds in terms of G^T still include some nuclear physics aspects which may change, through not dramatic, bounds given in terms of two body amplitudes.

Alternatively, bounds on effective T-odd P-even NN interactions may be

related to observables from T-odd P-odd experiments, assuming that the P-odd part is due to the standard model. A rough estimate of the corresponding limit on the T-odd P-even part f^T in the electric dipole moment of the neutron would be $|f^T| < 2 \times 10^{-5}$ [14,38]. From this number implications on the limits on a generic T-odd P-even meson nucleon coupling strength g^T_{MNN} may be found. An educated "guess" gives $|g^T_{MNN}/g^E_{MNN}| = \phi < 10^{-4}$ [14]. Using essentially the same experimental input Khriplovich argues that one might "expect" a bound of $| \left\langle t^T_{NN} \right\rangle | < 2 \times 10^{-6} | \left\langle t^E_{NN} \right\rangle |$ [38]. The large discrepancy between these two numbers is mostly due the implicit mass scale, introduced to express the bound. Khriplovich relates the coupling strength to the small pion mass, whereas Herczeg's result accommodates for a large mass responsible for a short range effective T-odd P-even NN interaction [14]. Therefore, in order to compare the number given in [38] with the results for the pd scattering given below, it is necessary to renormalize the mass scale and introduce an additional factor ($m_A^2/m_\pi^2 \simeq 90$) with m_A the mass of the axial vector meson given below. One then finds $| \left\langle t^T_{NN} \right\rangle | < 2 \times 10^{-4} | \left\langle t^E_{NN} \right\rangle |$, in accordance with ref. [14].

A general parity and time reversal symmetry conserving as well as rotational and isospin invariant amplitude t^E_{NN} for two nucleons may be written in terms of a Wolfenstein type parameterization [34], i.e. in the cm frame

$$\begin{aligned} t^E_{NN} &= a + b\underline{\sigma}_i \cdot \underline{\sigma}_j + c\, i(\underline{\sigma}_i + \underline{\sigma}_j) \cdot \mathbf{q} \times \mathbf{p}/m_p^2 \\ &+ e(\underline{\sigma}_i \cdot \mathbf{p}\underline{\sigma}_j \cdot \mathbf{q} - \underline{\sigma}_i \cdot \underline{\sigma}_j \mathbf{q}^2/3)/m_p^2 \\ &+ d(\underline{\sigma}_i \cdot \mathbf{p}\underline{\sigma}_j \cdot \mathbf{q} - \underline{\sigma}_i \cdot \underline{\sigma}_j \mathbf{q}^2/3)/m_p^2 \end{aligned} \tag{17}$$

with momenta $\mathbf{q} = \mathbf{k}' - \mathbf{k}$, $\mathbf{p} = (\mathbf{k}' + \mathbf{k})/2$ and m_p the proton mass introduced for dimensional reasons. The functions a to e are functions of Lorentzscalars, in the cm system given by $\mathbf{k}^2$, $\mathbf{k}'^2$ or $\mathbf{k} \cdot \mathbf{k}'$. In the following the first two terms are denoted as scalars, the third term as spin-orbit term, and the last two terms as tensor terms. Note, that for $\mathbf{k} = \mathbf{k}'$ the amplitudes a, b and e are respectively related to the amplitudes $\mathcal{F}^E_{00;0}, \mathcal{F}^E_{11;0}, \mathcal{F}^E_{11;2}$, defined above.

A generic time reversal violating but parity conserving amplitude t^T_{NN} may be written as follows (see also ref. [37])

$$\begin{aligned} t^T_{NN} &= f\underline{\sigma}_i \cdot \underline{\sigma}_j \mathbf{q} \cdot \mathbf{p}/m_p^2 + g\underline{\sigma}_i \times \underline{\sigma}_j \cdot \mathbf{q} \times \mathbf{p}/m_p^2 \\ &+ h(\underline{\sigma}_i \cdot \mathbf{p}\underline{\sigma}_j \cdot \mathbf{q} + +\underline{\sigma}_j \cdot \mathbf{p}\underline{\sigma}_i \cdot \mathbf{q} - 2\underline{\sigma}_i \cdot \underline{\sigma}_j \mathbf{p} \cdot \mathbf{q})/m_p^2 \end{aligned}$$

$$+g'(\underline{\sigma}_i - \underline{\sigma}_j) \cdot \mathbf{q} \times \mathbf{p}/m_p^2(\underline{\tau}_i \times \underline{\tau}_j)_0 \tag{18}$$

The functions f, g, g' and h are again functions of Lorentzscalars. The first term is of scalar type, the second depends on spins and angular momentum, the third is of tensor type (with arbitrary dependence on isospin), and the last term is of isovector spin-orbit type, included to account for a possible violation of C-symmetry. Note, that the T-odd P-even amplitude vanishes for $\mathbf{k}' = \mathbf{k}$, which reflects the results for $j_1 = j_2 = \frac{1}{2}$given above.

Possible NN potentials for the T-odd amplitude of eq. (18) are provided through (ρ-meson) vector exchange, with a C-violating (and hence T-violating) isospin dependence [18] or through axial vector exchange [18,39], reflecting the different behavior of axial vector and pseudo tensor interactions under time reversal symmetry, respectively. They are given in the following.

The vector exchange potential reads in momentum space, up to order p^2/m_p^2 [18]:

$$v_\rho^T = i\phi_\rho \kappa_\rho g_{\rho NN}^2(\mathbf{q}^2)/[8m_p^2(\mathbf{q}^2 + m_\rho^2)](\underline{\sigma}_i - \underline{\sigma}_j) \cdot \mathbf{q} \times \mathbf{p}(\underline{\tau}_i \times \underline{\tau}_j)_0 \tag{19}$$

The strength $\phi_\rho = g_{\rho NN}^T/g_{\rho NN}$ denotes the strength of the T-violating potential relative to the T-conserving one. The other parameters are taken from ref. [40] Table 5, viz. $m_\rho = 0.769\text{GeV}$, $\kappa_\rho = f_{\rho NN}/g_{\rho NN} = 6.1$, $g_{\rho NN}^2(\mathbf{q}^2 = m_\rho^2)/4\pi = 0.81$, and the $\mathbf{q}^2$-dependence of the strength is given by the form factor

$$F_{\mu NN}(\mathbf{q}^2) = [(\Lambda_\mu^2 - m_\mu^2)/(\Lambda_\mu^2 - \mathbf{q}^2)]^{n_\mu} \tag{20}$$

with $\Lambda_\rho = 2\text{GeV}$ and $n_\rho = 2$.

The axial vector exchange potential reads in momentum space, up to order p^2/m_p^2:

$$v_A^T = i\phi_A g_{ANN}^2(\mathbf{q}^2)/[8m_p^2(\mathbf{q}^2 + m_A^2)](\underline{\sigma}_i \cdot \mathbf{p}\underline{\sigma}_j \cdot \mathbf{q} + \underline{\sigma}_j \cdot \mathbf{p}\underline{\sigma}_i \cdot \mathbf{q} - \underline{\sigma}_i \cdot \underline{\sigma}_j \mathbf{q} \cdot \mathbf{p}) \tag{21}$$

with $m_A = 1.26\text{GeV}$ axial vector meson mass. The isospin dependence is not restricted and may be isoscalar, isovector or isotensor. The coupling strength g_{ANN} is not well know empirically from NN potentials. One may choose $g_{ANN}^2(\mathbf{q}^2 = m_A^2)/4\pi = 3.8$ [39] for an isoscalar $\underline{\tau}_i \cdot \underline{\tau}_j$-dependence, with a cut-off similar to the one used in the Bonn potentials with $n_A = 1$

and $\Lambda = 2$GeV as for other heavy mesons [40]. The coupling constant would then be $g^2_{ANN}(\mathbf{q}^2 = 0)/4\pi = 1.56$, close to the values of the a_1-meson of the Virginia potential with meson nucleon coupling constants calculated from quark symmetry rules and quark models [41].

4. Results

The deuteron wave function used in the calculation is generated by the Bonn potential [42]. To simplify the calculation of spin matrix elements only S-waves are considered.

The potential to determine the functions a to e for the strong amplitudes in eq. (17) are provided through one boson exchange with the parameters of the Bonn potential [40]. However, for the present estimate the amplitude is evaluated in Born approximation only, viz. $t^E_{NN} \simeq v_E$ in eq. (16). This approximation may not be sufficient but simplifies the calculation considerably. However, any error in the strong transition amplitude would effect the T-odd observable only linearly, see eq. (15). A comparison with the T-even break-up data at the relevant energies shows, that a factor of three uncertainty for the transition amplitude, and hence the T-odd observable, is possible. A more elaborate analysis is certainly needed to reduce the number of possible uncertainties. Some uncertainties however may also be due to relativistic effects, which should not be negligible above momenta larger than 1 GeV. Part of that is taken into account by using relativistic kinematics instead of the nonrelativistic one.

Presently, only elastic and break-up channels are considered. Other final channels including particle production (e.g. of pions) are neglected. This is justified as long as the energies are below particle threshold (e.g. pion threshold). However open channels may become more important for higher energies. Fortunately, any additional contribution is in favor of the P-even T-odd observable considered, if no accidental cancellations occur at the energy chosen for the experiment.

The first model (model I) considered is a estimate of the bounds that might be expected from a pd-forward scattering experiment. To this end the

unknown functions f, g, h and g' in eq. (18) are taken constant. For a very short range interaction and moderate energies this may be justified, since the deuteron wave function cuts out contributions of higher momenta in the integrals and therefore the momentum dependence is not so important.

For the elastic channel the isospin independent amplitudes dominate. However, the contribution of this channel to the total spin correlation coefficient is negligible, i.e. two orders of magnitude smaller than the one induced by the break-up channel, which will be discussed in the next paragraph. The charge dependent amplitude would contribute only via charge symmetry breaking transitions and therefore would lead to a suppression of the amplitude by the size of charge symmetry breaking, i.e. $\simeq 10^{-3}$ see e.g. [43].

Contributions due to the break-up channel of model I are shown in fig. 1. Since the total phase is unknown only absolute values are shown. The largest contribution is a tensor T-odd amplitude ($\propto h$ in eq.(18)) with a scalar T-even rescattering ($\propto b$ in eq. (17)), generated by π, ρ exchange. Other contributions in this simple estimate are roughly one order of magnitude smaller.

To obtain the total spin correlation $\mathcal{S}^T$, one needs to know the unpolarized *total* cross section σ^0_{tot}. Unfortunately not all relevant energy values are deducible from experimental data [36]. There is a gap between $k_{cm} \simeq 0.2$GeV and $k_{cm} \simeq 0.5$GeV. In fact, as seen in both figures this is the region where the maxima of the break-up part of $\mathcal{S}^T\sigma^0_{tot}$ occur. Provided a measurement of σ^0_{tot} in that region leads to $\sigma^0_{tot} \simeq 50$mb, which is the value at $k_{cm} \simeq 0.5$GeV [36], and provided an accuracy of 10^{-6} as mentioned in the introduction, a rough bound on $\left\langle t^T_{NN} \right\rangle$ could be achieved from model I to be $|\left\langle t^T_{NN} \right\rangle| < 2 \times 10^{-4} |\left\langle t^E_{NN} \right\rangle|$.

The second approach (model II) leads to bounds on coupling strength rather than to bounds on matrix elements. This is one step further in the analysis. To this end, the unknown functions f, g, g' and h of eq. (18) need to be related to the T-violating potentials given in eq. (19) and (21). Due to the weakness of the T-odd interaction the two body amplitude will be evaluated in Born approximation, i.e. $t^T \simeq v^T$ in eq. (16). Therefore, the T-violating potentials does not give terms proportional to g.

In this framework the dominant contribution relevant for elastic proton deuteron scattering comes from the tensor part of an axial vector (f_1 -meson, eq.(21)) T-violating interaction interfering with the spin orbit part of a (ω-, eff. σ-meson) T-conserving rescattering interaction. Isospin dependent meson exchange contributes only via charge symmetry breaking. Nevertheless, as in the more general case, the elastic contribution can be neglected compared to the break-up contribution.

For the break-up channel the dominant contribution in model II is through charged meson exchange potentials. This is due to the spin orbit part of a T-violating vector exchange ($\rho^{\pm}$ meson, eq. (19)) with a tensor ($\pi^{\pm}$-, $\rho\pm$-meson) T-conserving rescattering interaction. The result is shown as long dashed line in fig. 2. Since the T-odd coupling strength ϕ_ρ is not known $\mathcal{S}^T\sigma^0_{tot}$ is given in units of ϕ_ρ. Other lines represent contributions due to the T-violating axial vector exchange with a T-conserving rescattering contribution induced by π, ρ exchange. The solid, dashed and dotted line are due to tensor (scalar), scalar (tensor) and tensor (tensor) T-violating (T-conserving rescattering) interaction. The total spin correlation $\mathcal{S}^T$ is given in units of the unknown strength ϕ_{a_1}.

The suppression of the solid and dashed lines in fig. 2 (model II) compared to fig. 1 (model I) is partly due to a factor 1/3 (Clebsch-Gordan-coefficient) arising from the evaluation of the scalar contribution. Note also, that the results of model II drop faster with momentum transfer than those of model I. This is, since short range correlations are treated more properly in model II due to the form factor cut off. In addition, not all possible contributions of model I are covered by the more microscopic approach of model II, since the meson exchanges are considered in Born approximation only.

The maxima of the T-odd asymmetry in fig. 2 appear at $k_{cm} \simeq 0.3$GeV. They are $\mathcal{S}^T\sigma^0_{tot} \simeq 0.04\phi_\rho$mb for ρ-exchange, and $\mathcal{S}^T\sigma^0_{tot} \simeq 0.02\phi_{a_1}$mb for a_1 exchange. Provided the experimental accuracy of 10^{-6} and $\sigma^0_{tot} \simeq 50$mb, a bound on ϕ_ρ may be achieved of $\phi_\rho = g^T_{\rho NN}/g_{\rho NN} < 10^{-3}$ and a bound on ϕ_{a_1} of $\phi_{a_1} = g^T_{ANN}/g_{ANN} < 2 \times 10^{-3}$. Note, that these bounds are on T-odd coupling constants and not on matrix elements $\left\langle t^T_{NN} \right\rangle$.

5. Discussion and Conclusion

Firstly, it is important to note that microscopic models for a parameterization of T-odd P-even forces lead to nonzero contributions to pd forward scattering amplitude. This result is, presumably, not affected by final state interaction effects. The reason is that it is not a phase measurement but measures the total T-odd cross section. Indeed, using a different terminology, final state interaction is responsible to exhibit the generic T-odd effect.

Also, Coulomb interaction does not lead to divergences for the T-odd spin correlation. This is due to the necessary interference of the Coulomb force with a T-odd interaction. Therefore, Coulomb-interaction occurs only linearly in the observable. Due to the weakness of the electromagnetic coupling constant, Coulomb interaction can be neglected in the present treatment.

In this paper I present a first estimate on the bounds of T-violating P-even potentials that might be expected from a pd forward scattering experiment. It seems that the favored cm momentum to conduct an experiment is around $k_{cm} \simeq 0.3$GeV/c ($k_{lab} \simeq 0.5$GeV/c). However, not all possible T-odd channels have been taken into account so far, i.e. the pion production channels are neglected. Conclusions might change qualitatively, if those channels would dominate the T-odd observable; in particular higher energies might become more preferable. In order to reduce possible uncertainties a next step should incorporate the complete two body t-matrix of strong interaction. Some relativistic effects have been taken into account by using relativistic kinematics. However, lacking a covariant theory of three interacting particles the analysis at even higher energies may become more difficult. In addition, the total cross section needs to be known experimentally in the energy region where the T-odd observable has its maximum. It may also be advantageous to investigate other spin-1 nuclei than the deuteron, which may lead to enhancement through collective effects, even if the polarization of such nuclei is more delicate.

Although the theoretical analysis can still be improved, it is already possible to conclude that carrying out a forward scattering experiment to test time reversal symmetry is highly desirable. It will provide a direct measurement of bounds on T-odd P-even NN amplitudes of $|\left\langle t^T_{NN}\right\rangle| < 2\times10^{-4}|\left\langle t^E_{NN}\right\rangle|$ with an accuracy compatible to the measurements of electric dipole moments. It has been shown that due to the relatively simple system the bounds that may be reached on that fundamental symmetry can directly be related to

bounds on coupling constants of generic effective T-violating P-conserving model NN interactions and would lead to $\phi < 10^{-3}$.

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Figure Caption

Figure 1

Model I: Total spin correlation $|\mathcal{S}^T \sigma^0_{tot}|$ due to the break-up channel: solid line contributions due to $\langle hb \rangle$, dashed line due to $\langle fd \rangle$, long dashed line due to $\langle g'd \rangle$, dashed dotted line due to $\langle gc \rangle$, dotted line due to $\langle hc \rangle$. See eqs. (16,17,18).

Figure 2

Model II: Total spin correlation $|\mathcal{S}^T \sigma^0_{tot}|$ due to the break-up channel: long dashed line ρ exchange, all other lines due to a_1 exchange T-violating P-conserving potential. Solid line scalar(tensor) dashed line tensor(scalar), dotted line tensor(tensor) T-violating interaction (T-conserving rescattering). Results are normalized to the T-odd strength parameter ϕ_ρ, ϕ_{a_1}, resp.

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